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## **Optimal Vehicle Size, Haulage Length and the Structure of Distance- Transport Costs**

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### *Abstract*

This paper deals with two interrelated questions, namely the optimum size of a vehicle or vessel, and the structure of transport costs with respect to haulage distance. The relationship between these two questions has not been coherently dealt with previously in the literature, and the heuristic attempts at doing so, have lead to both theoretical and empirical anomalies. This paper adopts an inventory optimisation approach in order to show that both of these questions can be treated in a unified manner and this allows us to show, first, that under very general conditions the optimum size of a ship increases with the haulage distance and haulage weight, and second, that the observed structure of transport costs with respect to the haulage distance and quantity is itself a result of this optimisation problem.

## 1. *Introduction*

It is a well-known empirical observation that transport rates normally taper with increasing haulage distance, as described in Fig 1., thereby producing a generally concave relationship between transport costs and haulage distance. At the same time it is also observed that in many cases, transport rates fall with respect to the haulage quantity for any given distance, as described by Fig.6, thereby producing a convex relationship between transport rates and the quantity shipped. These observations are normally termed *economies of distance* and *economies of scale* in transportation, respectively. For many transportation models such observations are simply taken as assumed and any analysis proceeds on the basis of these assumptions. However, exactly why transport rates should behave in this way is actually not at all clear from an analytical point of view. The problem appears initially when we try to relate the observed structure of distance-transport costs to the costs of moving vehicles or vessels, as part of the overall problem of determining the relationship between the optimum size of a vehicle or vessel and variations in haulage distance. Under conditions where observed transport costs taper with haulage distance, there is no formal analytical proof that the optimum size of a vehicle or vessel (from hereon referred to as ‘vehicle-vessel’), increases with haulage distance (Thorburn 1960; Jansson and Shneerson 1982; 1987). Although we normally assume this fact, as we will see shortly, the existing heuristic attempts at accounting for this can be shown to be analytically indeterminate. The reason for this is that while the outcome of the vehicle-vessel size optimisation problem depends on the behaviour of distance-transport costs, the behaviour of distance-transport costs can also be shown to depend on the outcomes of the vehicle size optimisation problem (Bacon 1984, 1992; 1993; McCann 1993, 1998; McCann and Fingleton 1996). The result of this is not only that a formal proof of the relationship between optimal vehicle-vessel size and haulage distance has not previously been provided, but also that no theoretical explanation as to why transport costs are concave with respect to haulage distance has yet been given.

The aim of this paper is to provide a general proof to both of the above problems. In order to do this, the derivation of the structure of transport costs and haulage distance will be discussed within the overall framework of shipment size optimisation theory. Our purpose here is not to question the validity of standard shipment or vehicle-vessel size optimisation theory. Rather, we show here that both of these questions, namely that of the optimum vehicle-vessel size, and that of the non-linear structure of transport costs with respect to haulage distance, are in fact the same problem. Moreover, they can be solved within the existing framework by unifying them into a single consistent theoretical approach, which avoids the anomalies inherent in previous analyses. This approach allows us to prove the very general conditions under which the optimum size of a vehicle-vessel does increase with the haulage distance or the haulage weight. Furthermore, it also allows us to prove that transport rates are almost always concave with haulage distance irrespective the number of vehicle-vessel choices we have or the returns to scale that they exhibit. The reason is that the observed concave structure of distance-transport costs, is itself endogenously generated by the derivation of the optimum size of a shipment or vehicle-vessel.

The paper is organised as follows. In section two, we will outline how these issues have previously been dealt with in the literature and indicate the analytical problems that these explanations create. In section 3, we will discuss the relationship between two different analytical specifications of the shipment size optimisation problem. The first approach specifies transport rates as a function of the distance and quantity of the bundle of goods to be shipped, while the second approach specifies transport rates in terms of vehicle-vessel haulage costs. We will see that at present these two approaches do not give us consistent analytical results to the problem of the relationship between the optimum shipment size and the haulage distance. Therefore, it is necessary to unify the approaches to the specification of transport rates in these two models in order to produce coherent results. In section 4 this is done initially for the case of a single vehicle-vessel. In section 5 this unified approach is then extended to the more general case where we have multiple vehicle-vessel choices. In section 6 we will provide a formal proof of the conditions under which the vehicle-vessel size increases with the haulage distance. This will also allow us to prove that transport rates are almost always concave with the haulage distance irrespective of whether or not we experience economies of scale in vehicle-vessel hauling costs, and irrespective of the number of vehicle-vessel choices we have. The reason for this is due to the calculation of the optimum shipment size itself. Finally, we will be able to explain the conditions under which haulage costs are convex with respect to the haulage weight.

## 2. The Problem

Within transport economics, there is as yet no formal analytical proof of the conditions under which the optimum size of a vehicle-vessel increases with haulage distance. The problem centres around the well-known paradox that observed freight rates per ton normally taper with haulage distance as in Fig 1., whereas the observed cost per ton of a given vehicle or vessel is generally linear with distance, as described by Fig.2.

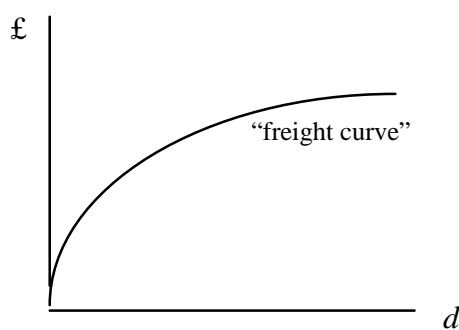


Fig. 1

Freight rate per ton tapering with distance

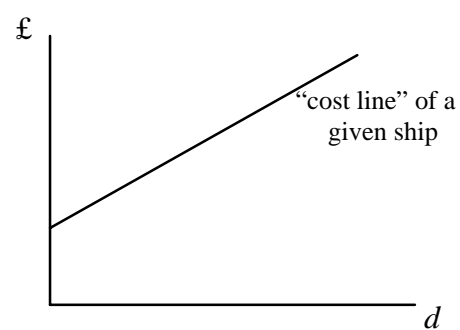


Fig. 2

Costs per ton as a linear function of distance

The existing ‘indirect proof’ by Thorburn (1960) in the case of water-borne shipping has been described by Jansson and Shneerson as “a possible point of departure” (1982 p.226) and “a useful diagrammatic technique” (1987 pp.144-145). The same argument was also employed by Alonso (1964) in the case of road and rail shipments. Thorburn’s heuristic approach is explained by Jansson and Shneerson thus:

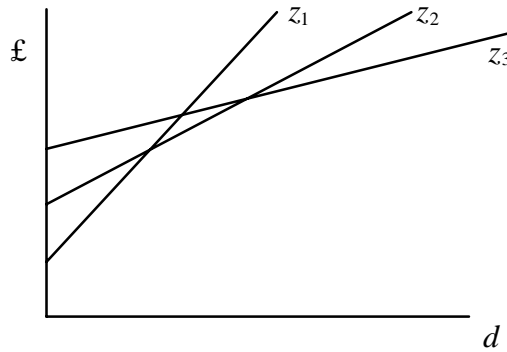


Fig. 3  
Costs per ton for different ships as a  
function of distance

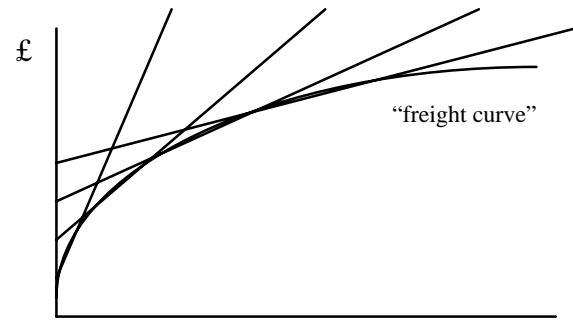


Fig.4  
The freight curve as an envelope to  
cost lines of given ships

When we compare the cost per ton functions of various alternative vessels of increasing size,  $z_1, z_2, \dots, z_n$ , if we assume that the port or terminal handling costs per ton, represented by the positions of the intercepts, and the vehicle-vessel hauling costs per ton, represented by the function slopes, behave as described according to Fig.3, Jansson and Shneerson (1982; 1987 pp144-145), argue that we can draw an envelope function as described in Fig 4. Observation of the point of tangency between a particular vehicle-vessel function and the envelope suggests that a larger vehicle-vessel will be used for longer routes. However, there are two fundamental problems with this heuristic approach.

(i) The theoretical validity of such an analysis depends crucially on the there being both a systematically positive empirical relationship between the size of an individual vehicle-vessel and its terminal or port handling costs, and a systematically negative empirical relationship between the size of an individual vehicle-vessel and its hauling cost per ton. Otherwise, it would not be possible for us to draw Fig.4. Moreover, these systematic relationships would also need to hold empirically in the case where we have vehicle-vessels of different vintages, in spite of the fact that technological improvements which reduce the port or terminal handling costs and movement-hauling costs will be embodied in newer vehicle-vessels. In the case of marine shipping, the empirical evidence of Heaver and Studer (1972), Robinson (1978) and Garrod and Miklius (1985) indicates that no such systematic functional relationships as described in Fig.3 necessarily exist. Indeed, if Fig.3 was altered to take account of their findings, the optimum vessel size on all routes would be extremely large, as Garrod and Miklius (1985) correctly point out.

(ii) It is not possible to use observed *cost per ton* freight rates with respect to distance and quantity of the bundle to be shipped, as exogenously determined inputs to the problem of the optimum vehicle-vessel size. The reason is that such observed freight rates themselves depend on the fleet management principles employed by third-party hauliers using their own fleets of vehicle-vessels. The result is that such an approach cannot tell us how the optimum size of a *vehicle-vessel* is related to haulage distance. All it can tell us is how the optimum size of a *shipment*, belonging to a customer who does not make decisions concerning the optimal employment of vehicle-vessels, is

related to haulage distance. Yet, as we will see in the next section, these are analytically two quite different problems, and it is these two problems which need to be unified.

In order to unify these two approaches analytically it is first necessary for us to compare how shipment optimisation problems are solved:

- (a) with transport costs specified as exogenously given freight rates dependent on the quantity and distance of the bundle of goods to be shipped, as in Fig.1, and
- (b) with transport costs specified as endogenously dependent on vehicle-vessel movement costs, but independent of the quantity and distance of the bundle of goods to be shipped as in Fig.2.

To do this we must specify transport costs in terms of an explicitly spatial measure such as transport *cost per ton-mile*, as is done in spatial pricing models, in order to allow the endogenous nature of transport cost generation to be investigated. Then it is necessary to see how these two analytical approaches can be unified initially in the case of a single vehicle-vessel. Subsequently we will be able to extend the analysis to the case of multiple vehicle-vessel choices.

### 3. *Shipment Size Optimisation Models*

The question of the determination of the optimum size of a shipment and the associated question of the optimum frequency of a shipment is a well-rehearsed problem. Here we compare two types of approaches to such problems which explicitly include distance as a variable.

#### (a) *Models with transport costs related to the quantity and distance of the bundle shipped*

Models in which transport costs are related to the quantity and distance of the bundle shipped are the usual type in spatial pricing models. In terms of shipment size optimisation problems these can be represented by the following approach:

For a shipper which ships a quantity  $m_i$  per time-period of inputs of f.o.b. source price  $c_i$  per unit of  $m_i$ , over a distance of  $d_i$  at a per ton-mile transport cost  $t_i$ , the total transportation costs involved can be represented as

$$t_i d_i m_i$$

and the delivered c.i.f. price can be represented as:

$$c_i + t_i d_i$$

which represents the value of an individual unit of inventory if the firm is a bonded carrier. Alternatively, if the shipper is simply a carrier, then the inventory time costs contribution of the fuel used are given by

$$t_i d_i^1$$

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<sup>1</sup> The time cost element of fuel used in transit was recognised by Jansson and Shneerson (1982). Our model combines both the fuel-time and the inventory time costs within the same inventory framework, whereas they were treated separately by Jansson and Shneerson (1982) and Garrod and Miklius (1985), respectively.

Assuming that goods are delivered and consumed at a constant rate and there are no stock-outs or shortages,<sup>2</sup> it is initially possible to represent the total *logistics* costs per time-period faced by the firm as (McCann 1993;1996):

$$\text{Total Logistics Costs } TL C_i = \underbrace{\frac{m_i S_i}{Q_i}}_{\text{Port Costs}} + \underbrace{\frac{I Q_i (c_i + t_i d_i)}{2}}_{\text{Inventory Holding Costs}} + \underbrace{t_i d_i m_i}_{\text{Transport Movement Costs}} \quad (1)$$

where  $S_i$  represents any fixed component of port or terminal-handling costs which are independent of the capacity of the vehicle-vessel, but are incurred every time a vehicle-vessel berths,  $I$  is an inventory holding cost coefficient, usually represented as the sum of the interest and insurance rates, and  $Q_i$  is the shipment size of the goods or people (Findlay 1983). Transport movement costs are the fuel and labour-time costs incurred in moving the goods or people between ports of call. From this we can determine the optimum shipment size thus:

$$\frac{\delta(TL C_i)}{\delta Q_i} = \frac{-m_i S_i}{Q_i^2} + \frac{I(c_i + t_i d_i)}{2} = 0 \quad (2)$$

and:

$$Q^* = \sqrt{\frac{2m_i S_i}{I(c_i + t_i d_i)}} \quad (3)$$

From equation (3) we see that the optimum shipment size is a function of the total transport costs per ton of cargo carried  $t_i d_i$ . The particular relationship between the optimum shipment size and the haulage distance will depend on exactly how total transport costs change with haulage distance. In this case, it would initially appear that as long as total transport costs are a positive function of distance, the optimised shipment size  $Q_i$  always falls as haulage distance  $d_i$  increases. Clearly, this result is counter-intuitive, particularly in the light of the empirical evidence (Kendall 1972; Jansson and Shneerson 1987). As we will see in the next section, however, the reason we observe this incongruous result is that there is a fundamental analytical problem with the this approach. This is because in shipment size optimisation problems, transport rates  $t_i$  in reality will also be a function of the shipment bundle size  $Q_i$ , while  $Q_i$  is a function of  $t_i$  (McCann and Fingleton 1996). The result is that we have too many variables and the problem becomes indeterminate (Bacon 1984). This is the reason why spatial pricing techniques have previously been unable to provide any analytical solution to the problem discussed in section 2. As Bacon (1984 p.115) points out “the form of the transport cost function plays a key role once we allow frequency to become a variable. Models in which transport costs are proportional to bundle size ...yield completely unsatisfactory results, and must be abandoned for

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<sup>2</sup> Variable demand and one-off deliveries also imply that a variety of individual vessels or vehicles will be needed at different times. Variable demand uncertainties are usually accounted for by calculating the optimum buffer stock to add to the simple optimum shipment quantity (Love 1979) and the existence of these buffer stocks does not fundamentally alter the behaviour of our simple model. The economic shipment principle will still be used to determine the general logistics arrangements, while one-off shipments will be organised individually. This means that although our model is set in a steady-state context, its conclusions are not fundamentally altered by uncertainty.

models where transport cost is fixed with respect to bundle size". This leads us to the second type of shipment size optimisation models.

(b) *Models with transport costs defined as vehicle-vessel movement costs*

The simplest model in which the optimum size of a shipment is determined with respect to haulage distance, in the case where transport costs are specified as vehicle-vessel moving costs, is the case of a single vehicle-vessel. Subject to the constraint that the calculated optimal value of  $Q$ , given as  $Q^*$ , does not exceed the total capacity of the vehicle, the shipment size problem can be defined (Bunn 1982; Bacon 1984; 1992; 1993) as determining the shipment size  $Q$  which minimises:

$$TLC_i = \frac{m_i}{Q_i} (S_i + d_i v_i) + \frac{I Q_i c_i}{2} \quad (4)$$

where  $v_i$  represents the cost of moving the particular vehicle-vessel through a distance of one mile. In other words, every time the vehicle is moved through a particular distance  $d_i$  the total transport costs incurred are given as  $v_i d_i$ , as described by Fig.2.

Once again, taking the first-order conditions we have:

$$\frac{d(TLC_i)}{d Q_i} = -\frac{m_i}{Q_i^2} (S_i + d_i v_i) + \frac{I c_i}{2} = 0 \quad (5)$$

and:

$$Q_i^* = \sqrt{\frac{2 m_i (S_i + d_i v_i)}{I c_i}} \quad (6)$$

Observation of (6) suggests that in the case of a single vehicle-vessel, the optimised shipment size is a positive function of distance. However, this result cannot yet inform us concerning the question of the relationship between the optimum vehicle-vessel size and the haulage distance, because it is limited to the case of a single vehicle-vessel. Nor can this model throw any light on the observed structure of transport costs.

(c) *The Argument*

The key argument of this paper is that it is possible to combine models of type (a) with models of type (b) in order to provide analytical solutions to the problems outlined in sections 1 and 2. In order to do this we must be able to overcome the problem of indeterminacy generally faced by models of type (a), and to allow models of type (b) to be extended to incorporate a more general description of transport costs which allows for the possibility of choices between multiple vehicle-vessels. The first stage of this analytical problem is to overcome the problem of indeterminacy in models of type (a) by describing transport rates in terms of vehicle hauling costs, as in models of type (b). In section 4 this will be done initially in the case of one vehicle-vessel. Then in section 5 this principle will be extended to the case where the logistics or transport planner has multiple vehicle-vessels to choose from. In section 6, this more general

formulation of shipment size optimisation problems can then be employed to provide proofs to each of the problems concerning the relationship between the optimal vehicle-vessel and the haulage distance and weight, and also to prove that total transport costs will be a concave function of distance even for a single vehicle-vessel, or for multiple vehicle-vessels irrespective of whether they exhibit economies of scale.

#### ***4. A unified approach to the mathematical description of transport rates within shipment optimisation problems***

In order to arrive at a unified approach to discussing transport costs analytically within the shipment size optimisation framework it is initially necessary to discuss the behaviour of individual vehicle or vessel transport costs and to see how these are related to the frequency of shipments made.

In many spatial models, per ton-mile transportation costs are often held fixed for analytical simplicity. What this actually means is that the transport cost per ton-mile is independent of the absolute quantity of goods carried. As we see in McCann (1993; 1998), in the case of individual batch shipment deliveries, this situation cannot occur. Goods can be carried by transportation forms such as truck, train, ship, airplane etc. These truckloads, trainloads, planeloads etc. represent the upper limits of the potential delivery batch sizes. Once the volume of goods per shipment increases above these size limits, transport-movement costs will generally rise in a stepwise fashion, since the fixed overheads will rise. If however, we can assume for the moment that these shipment loads do not exceed the upper physical limits of the particular mode of transportation (Bunn 1982), we can for the moment assume that transportation costs are independent of volume carried (Bacon 1984, 1992, 1993). Using a specific form and size of transportation vehicle-vessel, such as a particular truck or ship, the cost of moving that vehicle-vessel over a unit distance  $d_i$  will be the cost of fuel consumed plus the labour-hours involved, including where appropriate the empty or ballast return journey. This movement cost per mile has already been defined as  $v_i$ .<sup>3</sup> Now we can assume for the moment that it makes negligible difference to the value of  $v_i$  whether the vehicle-vessel carries one unit of  $m_i$  over the distance  $d_i$ , or one hundred units of  $m_i$  over distance  $d_i$ . In other words, as long as the batch size  $Q_i$  does not exceed the physical size of the vehicle-vessel, the total cost of transporting the shipment  $Q_i$  over a unit distance is independent of the shipment size. The total movement haulage cost for each shipment  $Q_i$  will therefore be given by  $v_i d_i$ . What is evident, is that for a given vehicle-vessel, total transportation costs are a linear function of distance, as in Fig.2.

Now we can imagine two points a fixed distance apart  $d_i$ . One point is an input source and the other a market point, with a vehicle-vessel moving forwards and backwards between these two points shipping a quantity of goods  $m_i$  per time period from the input source to the market. The total distance traveled by this vehicle-vessel per time period is equal to  $f \times 2d_i$  where  $f$  is the frequency of shipment, i.e. the number of deliveries per time period. If the market requires 100 units of  $m_i$  per day,

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<sup>3</sup> This would be the equivalent of the parameter  $g$  in Bacon (1992, 1993) and  $k$  in Findlay (1983).



and if all units are delivered in one single consignment, i.e.  $Q_i = m_i = 100$ , the total movement costs per day are  $v_i d_i$ . If each unit of  $m_i$  is delivered individually, as in a pure JIT mechanism (McCann and Fingleton 1996), the total distance travelled per day by the ship is  $100 \times 2 d_i$  and the total transport costs are  $100 v_i d_i$ .<sup>4</sup> What we see is that as the delivery frequency increases for a fixed total quantity of material  $m_i$  to be shipped per time period, the total transport cost per unit of  $m_i$  carried also increases. In this particular case the ratio has increased from  $(v_i d_i / m_i)$  to  $(100 v_i d_i / m_i)$  as the number of deliveries per time period increased from one to one hundred. In the formulations we used earlier, the parameter  $t_i$  represents the transport cost per ton-mile i.e. per unit weight-per unit distance. In other words  $t_i$  represents the cost of transporting one unit of  $m_i$  through one unit of distance  $d_i$ . In the example above, in the first case where there was only one delivery per day, we saw that  $t_i = v_i / 100 m_i$  whereas in the second case where there were 100 deliveries per day,  $t_i = v_i / m_i$ . In other words, for a fixed total volume of material,  $m_i$ , the per ton-mile transport cost  $t_i$ , depends on the size of the individual delivery batch shipment  $Q_i$ . In this particular case where the carrying capacity of an individual vehicle-vessel is not exceeded, we have the general expression  $t_i = v_i / Q_i$ . The reason for this is that for any fixed geographical delivery distance a change in the delivery frequency implies that the total *shipment-distance* travelled per unit of  $m_i$  changes. In other words, the relationship between economic distance and geographical distance depends on the shipment frequency. Bacon (1984) therefore suggests that these issues should be dealt with sequentially, with location being the first choice and delivery frequency being the second choice. This is exactly the approach of the inventory and logistics optimisation techniques employed in reality.

From the above discussion we can reconcile the two shipment size optimisation approaches (a) and (b) by substituting the expression  $t_i = v_i / Q_i$  into our equation (1) to give:

$$TLC_i = \frac{m_i}{Q_i} (S_i + d_i v_i) + \frac{I Q_i c_i}{2} + \frac{I d_i v_i}{2} \quad (7)$$

which differentiating and setting gives equations (5) and (6) as before. The only difference is that (7) is actually more comprehensive than (4) in that it also includes the capital time costs of fuel and labour usage, as discussed in footnote 1. This means that for a single type of vehicle-vessel, we can convert a shipment size optimisation problem of type (a) into one which is of type (b), by substituting the expression  $t_i = v_i / Q_i^*$  into any model of type (a).

In the case of a single vehicle-vessel, in order to see how transport cost per ton-mile rates  $t_i$  typically behave we can observe how  $t_i = v_i / Q_i^*$  changes with respect to  $m_i$  and  $d_i$ . From equation (6) we see that the optimum delivery batch size increases with respect to the square root of  $m_i$ , and with respect to the square root of

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<sup>4</sup> A pipeline carrying a fluid has the same mathematical properties.

$d_i$ . Thus, assuming  $d_i$  is held constant, as  $m_i$  increases, the optimum delivery frequency  $m_i/Q_i^*$  will increase in proportion to  $\sqrt{m_i}$  and the total batch distance travelled per unit of  $m_i$  will fall with respect to  $1/\sqrt{m_i}$ . Therefore, the fall in  $t_i$  will be with respect to  $1/\sqrt{m_i}$  as  $m_i$  increases. This is the nature of quantity discounts. A similar line of reasoning can be used to analyse the impact on  $t_i$  of a change in the distance  $d_i$  between the input source and the location of the market. Assuming a fixed volume  $m_i$  of material is to be moved through a distance  $d_i$  per time period, we can see that as  $d_i$  increases, the optimum batch size will increase with respect to  $\sqrt{d_i}$ , the delivery frequency will fall with respect to  $1/\sqrt{d_i}$ , and consequently, so will the value of the parameter  $t_i$ . Thus the distance increase is somewhat offset by a rise in batch size. This is the nature of economies of distance. This fall in  $t_i$  will continue until the value of  $d_i$  is large enough to mean that all goods are shipped in a single batch per time period<sup>5</sup> beyond which there will be no further fall. The point here, is that even for a single vehicle which exhibits linear movement costs with respect to distance, as long as the carrying capacity of the vessel is not exceeded, the optimum shipment size means that the transport cost per ton-mile falls with respect to the square root of both the haulage distance. Therefore, under these conditions, for a given ton of shipment, the ton-mile transport cost and the transport cost per ton will both rise in proportion to the square root of the haulage distance, as has often been observed (Isard 1951; Tyler and Kitson 1987; Bayliss and Edwards 1970). This observation provides a partial reconciliation between the paradox suggested by Fig. 1 and 2, for the case of a single vehicle-vessel.

However, in order for this observation to become a general principle we must be able to show that this ‘square root law’ (Baumol and Vinod 1970) holds for transport rates when we also any incorporate port-terminal handling costs which are related to the size of a shipment, and also the question of multiple vehicle-vessel sizes. Variable port-terminal handling costs are incorporated into our model in a straightforward way in section 6 onwards<sup>6</sup>. The next section explains how the insights from this section into the relationship between the optimised shipment size and the structure of vehicle-vessel transport costs can also be generalised from the case of a single vehicle-vessel to the case of multiple vehicle-vessel choices.

## 5. Generalised Transport Rates with Multiple Vehicle-Vessel Choices

So far we have ignored is the question of the use of multiple vessel or vehicle types on the behaviour of transport rates within the shipment size optimisation problem. Indeed, the assumption that a transport or logistics planner is constrained to use a

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<sup>5</sup> At this point, the total costs of transportation become greater than the total inventory holding costs. This is a corner solution. As  $d_i$  is increased beyond this critical value, the frequency of delivery cannot fall any further and so the value of  $t_i$  cannot fall any further and will remain constant beyond this point. Thus, where all goods  $m_i$  are shipped in a single batch delivery,  $t_i$  will remain both constant and a minimum as  $d_i$  changes.

<sup>6</sup> See Appendix A.

particular vessel-vehicle for organising all goods shipments, which we have made up to now in order to employ the formula  $t_i = v_i / Q_i$  in our logistics model, seems to be very limited and inappropriate for most real world situations, given that logistics calculations normally involve fleet management considerations, in which the optimum size of the vehicle-vessel is the key question. However, we can adopt a similar approach as in section 4, but now extend it to the case of multiple vehicle-vessel sizes.

Where we have a range of vehicle-vessel types and sizes to choose from, we consequently also have a range of values of  $v_i$  which can be included into the calculation formulated in equations (4) to (7). Each different value of  $v_i$  will generate a different  $Q_i^*$  and for the firm the value of  $Q_i^*$  which is the most preferable is that particular value of  $Q_i^*$ , which over the range of vehicle-vessel choice alternatives, minimises the minimum total logistics costs calculated on the basis of any particular vessel type i.e. the *optimum optimorum*. If there are multiple vehicle-vessel types and sizes to choose from, each type with its own particular shipment capacity limit, the usual logistics approach is to calculate the optimum shipment frequency for each mode of transport, assuming that we are constrained to use a particular mode of transportation, and then to compare the relative minima of each of these vessel options. Such an approach is used to determine the general size, frequency, vehicle-vessel mode and size of individual deliveries, and one of the aims of the logistician will be to ensure maximum vehicle-vessel capacity utilisation, i.e. full load shipments of individual vehicle-vessels where at all possible, although as we have seen, full-load vehicle-vessel utilisation will not be the primary criterion governing the organisation of shipment deliveries.

We cannot know the value or behaviour of the parameter  $t$  in a type (a) model involving multiple vehicle-vessel choices without *a priori* knowledge of the values of all the other parameters. However, it is possible to provide a general principle.

Individual vessel or vehicle types often tend to exhibit shipment economies of scale. In other words, when we compare the relative costs of movement between different vessel or vehicle types and sizes, we see that the variation is normally much less than proportionate to the relative carrying capacities of the vessels (Jansson 1980). In other words, if we plot the per-mile movement costs  $v_i$  against the full-load carrying capacity  $Q_i$  for each vehicle-vessel, over a range of vehicle-vessel sizes and types,  $i=1 \dots n$ , in general we will see an upward sloping relationship as shown in Fig.5.

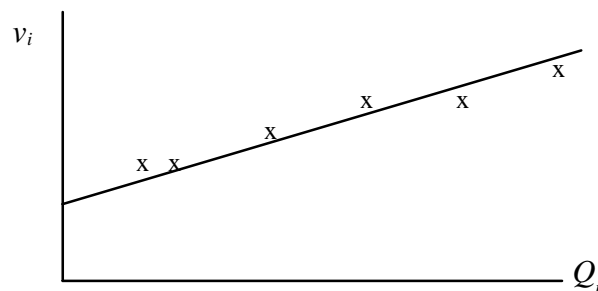


Fig 5  
The relationship between vessel movement costs and vessel carrying capacity

We can describe this relationship in terms of a regression function:

$$v_i = a + bQ_i$$

where  $a$  represents the positive intercept, given that the smallest vessel appropriate for industrial haulage purposes will generally incur non-zero movement costs, and  $b$  represents the positive slope, given that in general larger shipment carrying capacity vehicle-vessels will imply larger movement costs.

Now from our earlier discussion, if the logistics planner ensures that all individual shipments take place as full load shipments with respect to the capacity of the vehicle-vessels being employed across the range of vehicle-vessel choices available, such that the *optimum optimorum* is always achieved, we can represent a generalised transport rate function as:

$$t = \frac{a + bQ}{Q} = \frac{a + bQ^*}{Q^*} = \frac{a}{Q^*} + b$$

where  $a$  and  $b$  are the estimated intercept and slope parameter values across the range of vehicle-vessel choices available. This transport rate expression is much broader and more flexible than our previous expression because it allows for transport rates generated by shipment size optimisation behaviour within a choice set involving multiple vehicle-vessel sizes and types.

If we have a range of vehicle-vessel choices which exhibits economies of scale in individual shipment the gradient  $b$  will tend towards being zero with  $a$  being a non-zero intercept. Under these conditions the transport rate parameter  $t$  will tend towards being:

$$t = \frac{a}{Q} = \frac{a}{Q^*}$$

over the range of different vehicle-vessel types as well as for a single vehicle-vessel type, as had been represented by our earlier expression  $t_i = v_i / Q_i$ . The greater is the value of  $a$  and the smaller is the value of  $b$ , the more significant are the static economies of scale in shipment transportation. Alternatively, if our choice range of vehicle-vessels happens by chance to experience constant returns to scale in shipment size, in our above expression this would be represented by:

$$t = \frac{bQ}{Q} = \frac{bQ^*}{Q^*} = b$$

and in Fig.5 this will be represented by a straight line from the origin with a positive slope of gradient  $b$ . However, as we see in the Appendix C, in discrete shipment operations, the presence of the port-terminal handling cost indivisibilities means that it is not possible to experience an absolutely fixed value of  $t=b$  even if we were to happen experience constant returns to scale in shipment transportation. The only case where  $t=b$  is where there are no port-terminal costs and where we also happen experience constant returns to scale in shipment transportation. Under these circumstances, all goods will be delivered in a continuous-flow/JIT manner as discussed in section 4, and all per ton-mile transport rates will be fixed and independent of the haulage distance and quantity.

This more general approach as to how the structure of transport rates in models of type (a) are generated by optimised shipment calculations from models of type (b), but

which also involve multiple vehicle-vessel choices, now allows us to discuss the question of the relationship between the haulage distance and the optimum vehicle-vessel size. In order to do this we simply need to calculate the optimum *shipment* size under this more general transport rate specification, given that  $Q_i = Q_i^*$  under conditions of vehicle-vessel full-load carrying capacity.

## 6. Proof of the Relationship between the Optimum Vehicle-vessel Size, the Haulage Distance and the Structure of Transport Rates

**PROPOSITION 1:** *The theoretical optimum vehicle-vessel size is not always positively related to the haulage distance, although empirically this is the usual result.*

### Proof of Proposition 1

If we incorporate space costs and our general description of transport costs into our initial total logistics costs expression for shipments given in equation (1) we have:

$$TLC_i = \frac{m_i S_i}{Q_i} + \frac{I c_i Q_i}{2} + \frac{I d_i (b + \frac{a}{Q_i}) Q_i}{2} + \frac{s_i Q_i}{2} + m_i d_i (b + \frac{a}{Q_i}) \quad (8)$$

which rearranges to:

$$TLC_i = \frac{m_i (S_i + a d_i)}{Q_i} + \frac{Q_i [I(c_i + b d_i + s_i)]}{2} + d_i \left( \frac{Ia}{2} + m_i b \right) \quad (9)$$

Once again, differentiation and setting to zero gives:

$$\frac{\delta(TLC_i)}{\delta Q_i} = -\frac{m_i (S_i + a d_i)}{Q_i^2} + \frac{[I(c_i + b d_i) + s_i]}{2} \quad (10)$$

which gives an optimised shipment size as:

$$Q_i^* = \sqrt{\frac{2m_i (S_i + a d_i)}{[I(c_i + b d_i) + s_i]}} \quad (11)$$

In order to determine how the optimised shipment size is related to the haulage distance we need to take the derivative of (11) with respect to haulage distance thus:

$$\frac{\partial Q_i^*}{\partial d_i} = -\frac{Ib [2m_i (S_i + a d_i)]^{1/2}}{2 [I(c_i + b d_i) + s_i]^{3/2}} + \frac{a 2^{1/2} m^{1/2}}{2 [2m_i (S_i + a d_i)]^{1/2} [I(c_i + b d_i) + s_i]^{1/2}} \quad (12)$$

What we see is that the behaviour of the optimised shipment size, and consequently the optimum vehicle-vessel size, with respect to haulage distance, depends on the relationship between the vehicle-vessel movement costs and the vehicle-vessel carrying capacity, as defined by the values of  $a$  and  $b$ .

In order for the optimum vehicle-vessel size to increase with the haulage distance, equation (12) must be positive. In other words:

$$\frac{a}{[(S_i + a d_i)]^{1/2} [I(c_i + b d_i) + s_i]^{1/2}} > \frac{Ib[(S_i + a d_i)]^{1/2}}{[I(c_i + b d_i) + s_i]^{3/2}}$$

i.e.

$$a[I(c_i + b d_i) + s_i] > Ib[(S_i + a d_i)]$$

which rearranges to give:

$$\frac{a}{b} > \frac{I S_i}{I c_i + s_i}$$

The relationship between the optimised shipment size and the haulage distance depends on the extent of the static economies of scale inherent in vehicle-vessel shipment capacity. In order to understand the general conditions under which the optimised vehicle-vessel size will increase or fall with the haulage distance, we can once again discuss the two cases of significant economies of scale in vehicle-vessel movement costs relative to carrying capacity, and the case of constant returns to scale in this relationship.

If we experience significant static economies of vehicle-vessel movement costs relative to carrying capacity, where  $a$  is highly positive and  $b$  is equal or close to zero as in the Type (b) example outlined above, larger shipments will incur less than proportionately increased fuel-time costs per shipment. In other words, as the geographical haulage distance increases, the shipper can reduce the fuel-time costs per unit of cargo shipped *both* by increasing the size of an individual shipment and by reducing the shipment frequency. What is important here is that it is not the geographical haulage distance which is important, but the economic batch-distance travelled per unit of cargo hauled, where the batch distance is defined as the geographical distance multiplied by the shipment frequency. Therefore, assuming that the chosen vehicle-vessels are always at full load  $Q_i$  capacity, such that  $Q_i = Q_i^*$ , we can see that the optimum size of a vehicle-vessel will increase with respect to haulage distance if there are static economies of scale in vehicle-vessel movement costs relative to vessel carrying capacities.

On the other hand, if vehicle-vessel movement costs experience more or less constant returns to shipment size, i.e.  $a$  is equal or close to zero and  $b$  is highly positive, as represented by the Type (a) model outlined above, bulk shipments will incur fuel-time and stock inventory costs directly proportional to the shipment size. Under these conditions, over any given geographical haulage distance the only advantage in having bulk shipments is the reduction in total port-handling costs, which are a multiple of the number of shipments per time period. However, as the geographical haulage distance increases, the total fuel-time inventory costs will also increase, for any given shipment size. The result of this is that the optimised shipment size actually falls, and the optimised shipment frequency actually increases, in order to

offset this increased cost, *ceteris paribus*. Therefore, the optimum vehicle-vessel size also falls.

Hypothetically, there is also a third possibility, and this is the unique case where:

$$\frac{a}{b} = \frac{I S_i}{I c_i + s_i}$$

Under these conditions, we see that the optimised shipment size, and consequently the optimum vehicle-vessel size is invariant with respect to the haulage distance. However, this situation is purely a chance occurrence, which will not continue to hold in the face of interest rate or product price changes.

There is good empirical evidence that vehicle-vessel transportation generally does indeed experience significant economies of scale in the essentially static relationship between vessel movement costs relative and vessel carrying capacities (Jansson 1980). The observed result of the existence of these vehicle-vessel economies of scale is that observed shipment sizes are empirically observed to increase with the haulage distance (Kendall 1972; Jansson and Shneerson 1982, 1987). However, as we have seen here, there is no analytical reason why this should necessarily be the case. This observed empirical result is simply the outcome of the optimisation process being carried under conditions of significant economies of scale in vehicle-vessel movement costs and carrying capacities.

*QED*

**PROPOSITION 2:** *Freight transport rates per ton are always concave with respect to the haulage distance, except for the unique case in which the optimised shipment size is invariant with respect to the haulage distance.*

The observed freight rates charged by hauliers to customers will comprise a profit mark-up on top of the total logistics costs involved in any haulage operation, and the size of this mark-up will depend on the competitiveness and contestability of the industry over the particular haulage route in question (Davies 1986). However, there is no reason why the level of this mark-up should be systematically related to the haulage length or haulage quantity over all shipment routes. The result of this is that observation of the structure of total logistics costs with respect to haulage distance and haulage quantity will explain the behaviour of observed transport rates with respect to haulage distance and haulage quantity.

#### Proof of Proposition 2

In order to see why observed transportation costs per-ton vary with respect to haulage distance and haulage quantity in the way that they do, from our preceding discussion it is necessary to discuss how optimised total logistics-transport costs per ton of cargo carried vary with respect to the haulage distance and the haulage quantity. To do this it is first necessary to substitute equation (11) into equation (9), to give a total logistics-

transportation costs expression and then to divide by  $m_i$  to give a cost per ton measure, thus:

$$TLC_i / m_i = 2^{1/2} m_i^{-1/2} (S_i + a d_i)^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2} + d_i \left( \frac{Ia}{2m} + b \right) \quad (13)$$

To observe the relationship between total logistics-transportation costs per ton and haulage distance we take the first and second derivatives of (13) with respect to haulage distance  $d_i$ :

$$\frac{\delta(TLC_i / m_i)}{\delta d_i} = \frac{a \left[ I(c_i + b d_i) + s_i \right]^{1/2}}{2^{1/2} m_i^{1/2} (S_i + a d_i)^{1/2}} + \left( \frac{Ia}{2m} + b \right) + \frac{Ib (S_i + a d_i)^{1/2}}{2^{1/2} m_i^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2}} \quad (14)$$

and:

$$\begin{aligned} \frac{\delta^2(TLC_i / m_i)}{\delta d_i^2} = & -\frac{a^2 2^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2}}{4 m_i^{1/2} (S_i + a d_i)^{3/2}} + \frac{Ia b 2^{1/2}}{4 m_i^{1/2} (S_i + a d_i)^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2}} \\ & + \frac{Ia b 2^{1/2}}{4 m_i^{1/2} (S_i + a d_i)^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2}} - \frac{b^2 I^2 2^{1/2} (S_i + a d_i)^{1/2}}{4 m_i^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{3/2}} \end{aligned} \quad (15)$$

Equations (14) and (15) are always positive and negative, respectively; i.e. equation (13) is strictly concave in distance, if either  $a$  or  $b$  is zero. However, the general conditions under which equation (13) is concave in distance is:

$$\frac{a^2 \left[ I(c_i + b d_i) + s_i \right]^2 + I^2 b^2 (S_i + a d_i)^2}{(S_i + a d_i)^{3/2} \left[ I(c_i + b d_i) + s_i \right]^{3/2}} > \frac{2Ia b}{(S_i + a d_i)^{1/2} \left[ I(c_i + b d_i) + s_i \right]^{1/2}}$$

thus:

$$a^2 \left[ I(c_i + b d_i) + s_i \right]^2 - 2Ia b (S_i + a d_i) \left[ I(c_i + b d_i) + s_i \right] + I^2 b^2 (S_i + a d_i)^2 > 0$$

and:

$$\left\{ a \left[ I(c_i + b d_i) + s_i \right] - Ib (S_i + a d_i) \right\}^2 > 0$$

This expression must always be true, except where:

$$a \left[ I(c_i + b d_i) + s_i \right] = Ib (S_i + a d_i)$$

i.e. when:



$$\frac{a}{b} = \frac{I S_i}{I c_i + s_i}$$

which is the same condition as for equation (12) to be zero. In other words, there is a unique case where the optimised shipment size and vehicle-vessel size is invariant with respect to the haulage distance, and where equation (13) must therefore also be linear with distance, given that the total shipment-distance travelled must be linearly related to the geographical distance. Apart from this unique chance case, however, equation (13) is always concave in distance, irrespective of whether we experience constant or increasing returns to scale in vessel movement costs relative to carrying capacity. The reason is that the total shipment distance travelled is a function of the square root of the geographic distance.

*QED*

**PROPOSITION 3:** *Freight transport rates per ton are always convex with respect to the total haulage quantity.*

**Proof of Proposition 3**

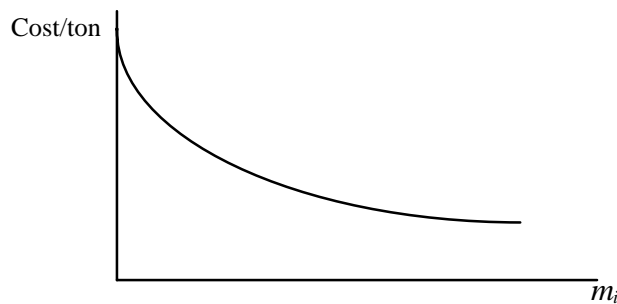
In order to observe the cost per ton relationship between total logistics-transportation costs and haulage quantity we take the first and second derivatives of (13) with respect to haulage distance  $m_i$  thus:

$$\frac{\partial(TL C_i / m_i)}{\partial m_i} = - \frac{(S_i + a d_i)^{\frac{1}{2}} [I(c_i + b d_i) + s_i]^{\frac{1}{2}}}{2^{1/2} m_i^{3/2}} \quad (16)$$

and:

$$\frac{\partial^2(TL C_i / m_i)}{\partial m_i^2} = \frac{3(S_i + a d_i)^{\frac{1}{2}} [I(c_i + b d_i) + s_i]^{\frac{1}{2}}}{2^{3/2} m_i^{5/2}} \quad (17)$$

Equations (19) and (20) which are negative and positive respectively, define a cost per ton relationship as described by Fig.6, in which the cost per ton falls with increasing shipment quantities per time period, although at a decreasing rate.



**Fig.6**

The variation in transport cost per ton with respect to the haulage quantity per time period.

This proof implies that as the total quantity per time period of freight to be shipped by an individual haulier over any given distance increases, the average freight rate falls with respect to the square root of the individual firm's market size. Given that the optimised shipment size is itself a square root function of the total shipment quantity per time period, this also implies that the cost per ton freight rate charged to individual customers will generally fall with respect to the square root of the individual customer's shipment size. However, there are many exceptions to this principle dependent on questions of service quality and scheduled freight consolidation practices, as discussed in Appendix B.

*QED*

Equations (14), (15), (16) and (17) indicate that the reason why observed transport costs behave in general as they do with respect to haulage distance and haulage quantity (Allen 1977; de Borger and Nonneman 1981) is due to the relationship between the *optimised* shipment and vehicle-vessel size, and the haulage distance and haulage quantity, rather than simply the static relationship between the movement costs, the port-terminal handling costs, and individual vehicle-vessel sizes. Moreover, this general relationship holds whether we have only a single vehicle-vessel or multiple vehicle-vessels to choose from, even allowing for the fact that transport rates will experience discontinuities and integer problems associated with limited vehicle-vessel choices.

## 7. Conclusions

Existing theoretical attempts at explaining the relationship between the optimum vehicle-vessel size, the haulage distance and haulage quantity have always foundered on the problem of how to incorporate the observed structure of transport costs into such analyses. This has meant that these questions have been dealt with in rather a heuristic manner, and when empirical observations are included in such models the results have been shown to be at best inconclusive. This paper has shown that it is possible to provide a consistent theoretical explanation of the previously paradoxical relationship between the optimum vehicle-vessel size, haulage distance, and the observed structure of transport costs, by combining standard inventory optimisation techniques with a model incorporating the scale relationships between vehicle-vessel shipment carrying capacities and vehicle-vessel movement costs. Under these assumptions, we see that the relationship between the optimum vehicle-vessel size and the haulage distance depends on the static relationship between vehicle-vessel movement costs and carrying capacities. At the same time, we see that the observed structure of transport costs with respect to haulage distance and quantity for each ton or ton-mile of cargo carried, is itself endogenously determined by the optimum vehicle-vessel size calculation, and not vice-versa, as has been described in some previous papers. By calculating the optimum shipment size on the basis of the movement costs of individual or multiple types of vehicles, transport costs per ton shipped will always be directly related to the square root of the haulage distance, and inversely related to the haulage quantity per time period, because of the impact of the haulage distance and haulage weight on the number of shipments made per time period for any given geographical distance, i.e. on the optimised shipment-miles distance, rather than simply on the geographical distance. Moreover, this general square root observation will hold even if we allow for the indivisibilities associated

with limited vehicle-vessel choices. The optimum vehicle-vessel size will also be seen to depend on the characteristics of the goods being shipped, with more valuable, bulky, perishable or fragile goods generally requiring more frequent shipments in smaller individual loads. This implies that the optimised total shipment distance, and consequently the absolute levels of the transport rates for these types of goods, will tend to be higher for any given geographical distance (Chisholm 1971), than for other goods. However, competition and contestability in transport markets still implies that the behaviour of transport rates with respect to the haulage distance and quantity will be as described here.

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### **Appendix A: Variable Port-Terminal Handling Costs**

For analytical simplicity, we have so far ignored the question of handling costs which are proportional to the size of the shipment, and have assumed a fixed port handling cost  $S_i$  which is independent of the size of the shipment. However, the existence of any such costs can be incorporated into the model quite easily. For a single type of good or mix of goods being moved by the shipper, it may be the larger the individual shipment, the greater the number of units of labour required to load and unload a vehicle-vessel in the terminal or port. Also, handling these larger shipments may incur greater land costs and the labour costs involving in inventory handling operations. In this case, the effect of these terminal-port handling costs due to larger shipments will be included in our framework via a space-handling coefficient term  $s_i$ .<sup>7</sup> On the variable  $Q$ . Once again, assuming that the demand for material remains constant over a time period, the total annual space costs incurred in holding inventory can be expressed as  $s_i Q_i / 2$  where  $s_i$  is the logistics space-cost coefficient defined as (McCann 1996; 1998):

$$s_i = 2 \times \frac{\text{bulk}}{\text{weight}} \text{ratio} \times [r R_i + w L_i]$$

and where:

$r$  = annual rent per square meter of warehouse/factory space

$w$  = annual unit wage of a warehouse/materials-handling worker

$R_i$  = area required to store one cubic meter of inventory of a particular product

$L_i$  = number of units of labour required to handle one cubic metre of inventory of a particular product.

Meanwhile, any variations in port-handling costs which are due solely to the characteristics of product being handled, will be reflected in variations in the values  $S_i$  and  $s_i$ , depending on whether these costs are independent or dependent of the size of shipment, respectively. In general, products which are fragile, perishable and/or bulky will have high values of  $S_i$  and  $s_i$ . In order to account for the effects of these costs on the optimum shipment size problem, it is simply necessary to also incorporate  $s_i Q_i / 2$  into equation (7) and then continue the optimisation procedure as described in section 6.

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<sup>7</sup>It may be that in some circumstances, newer, larger ships actually incur lower absolute levels of port-handling costs than some smaller ships. In our model, this will tend to reduce the value of the parameter  $s_i$  such that in some circumstances it may be that it tends towards being zero or even negative. However, as Garrod and Miklius (1985) point out, the role of the inventory capital costs will ensure that the optimum ship size is not infinitely large.

## **Appendix B:** *The Role of Consolidated shipments and Service Quality*

For long distance consolidated haulage, the haulier will employ large vehicle-vessels carrying large consolidated consignments, whereas for short distances, small vehicle-vessels carrying small consignments are used. Haulage contractors dealing with many clients will realise this principle by 'break bulk' logistics methods. Here, individual small deliveries from firms are picked up using various small vehicle-vessels, carrying small total shipments. These travel a short distance to a consolidation depot. These individual shipments are then grouped together to form a single very large consignment, which travels the long-haulage length to a similar distribution depot. From here, small individual consignments travel the short distances to the various customers. The vast majority of the haulage operation is carried out by having a large volume consolidated consignment batch travelling the long distance using a large vehicle-vessel, which will be calculated according to Economic Order Quantity principles described here. The structure of the logistics-costs faced by the haulage firm is the same as that of our logistics-costs expressions here, except that unless the firm is a bonded carrier, it will not pay capital finance costs on its inventory, only insurance plus space handling costs. In the case where the haulage firm has control over the logistics operation involving the consolidation of deliveries to and from a variety of firms, the total transport costs facing an individual customer firm rise in proportion to the square root of the haulage distance, but will be more or less linear with respect to the haulage weight. There are two reasons for this. First, where third party hauliers control the logistics operation, they organise a standard timetable of shipments according to the overall expected level of demand for their services between various locations per time period. The concern of the haulier here is to fill each individual vehicle shipment capacity so as to maximise the revenue per shipment, and so be able to charge as low a rate as possible. As the shipment contains goods from many firms, all individual units of goods will be charged the same per ton-mile rate as each other. Therefore, for any distance of haulage, the transport rate will be invariant with respect to the weight of material being moved for an individual client (Bayliss and Edwards 1970), except for discount. Similarly, in order to cover the insurance costs of their shipments the haulier will charge each customer a fixed rate per individual shipment per customer, which when consolidated with those from other customers, will cover the shipment insurance costs of the consolidated shipments (Chisholm 1971).

The second reason why haulage transport costs should be invariant with respect to the haulage weight of any particular customer is the quality of the delivery service (Allen and Liu 1995; Allen et al. 1985). The quality of the delivery service can be defined as the probability that any individual unit of good carried between two points will be delivered within a particular time period. If  $m$  rises unexpectedly, if  $Q^*$  is proportional to the square root of  $m$ , as  $m$  increases, the delivery frequency falls with respect to  $1/\sqrt{m}$ , and the transport cost per ton-mile falls in proportion to  $1/\sqrt{m}$ . What this means is that for a particular single unit of  $m$ , the probability of this being delivered within a particular time period falls in proportion to  $1/\sqrt{m}$ . In the case of a haulage firm carrying consolidated consignments containing individual units of  $m$  from many customers, in order to ensure that the particular service level between two points is maintained whatever the level of  $m$ , the delivery batch size must be proportional to  $m$  while still being proportional to  $1/\sqrt{d}$ . In this case, the transport cost per ton-mile will be constant for any level of  $m$ . However, this form of delivery

is only appropriate where the firm only has a small quantity of goods to be delivered to a particular place per time period. In the usual case where a firm consistently has a very large volume of materials to be moved per time period to a particular set of locations, it is more economical for the firm to have control over the logistics operation, irrespective of whether it is using ‘own account’ or third-party haulage services, and in practice, this is indeed the policy of most large firms.

**Appendix C:** *Proof that the transport rate parameter  $t$  cannot remain fixed even with constant returns to scale in shipment transportation.*

Under the hypothetical situation that we experience constant returns to scale in transportation, we would still not observe that  $t$  is fixed in a logistics-costs model calculated on the basis of the relationship between vehicle movement costs and vehicle carrying capacities, over a single or multiple vehicle-vessel types. The reason is that the assumption that  $t$  is fixed means that the relationship between vehicle-vessel movement costs  $v$  and carrying capacities  $Q_c$  is linear. However, within an Economic Order Quantity model, the assumption that  $t$  is fixed means that in general,  $v$  is a linear function of  $Q^*$ , rather than  $Q_c$ . Yet, this can never be the case. It is possible to see this simply by comparing any two types of vehicle-vessels, whose movement costs are  $v_a$  and  $v_b$ , respectively, where  $v_b = k v_a$  (for any positive constant  $k$ ), and whose carrying capacities are  $Q_a$  and  $Q_b$ , where  $Q_b = k Q_a$ . The relationship between these two vehicle-vessels exhibits constant returns to scale when we compare carrying capacity per shipment with the vehicle-vessel movement costs. Assuming that the logistics planner has a sufficient variety of vehicle-vessel types and sizes that we can assume there are no less than full-load shipments at the optimum arrangement, then we can set  $Q_a = Q_a^*$  and  $Q_b = Q_b^*$ . However, we see that constant returns to scale in vehicle-vessel shipment movement costs will not also imply that the transportation costs for any given haulage distance are fixed, i.e.  $t_a \neq t_b$ , because:

$$\frac{v_a}{Q_a^*} \neq \frac{v_b}{Q_b^*} \text{ given that } \frac{v_a}{\sqrt{\frac{2m(S + d v_a)}{Ic + s_a}}} \neq \frac{kv_a}{\sqrt{\frac{2m(S + d k v_a)}{Ic + s_a}}}$$

Moreover, even if by chance the per ton-mile transport rate happened to be equal for two different haulage shipment sizes over a given spatial haulage distance, it is clear that this result would not also hold over haulage distances in general, and that the transport rate would continually change. This point is important, because it means that we cannot write a spatial analytical model with a fixed per ton-mile transport rate  $t=b$  parameter as an assumption, as is often done, when we are discussing the question of frequency and shipment sizes, unless we have *both* no terminal costs per shipment and also no economies of scale in the relationship between vehicle-vessel movement costs and carrying capacities. Under these conditions all goods will be shipped in a continuous-flow/JIT manner in which  $t=b$  irrespective of the haulage quantity and haulage distance.